Configuration-space Faddeev calculation for nucleon-deuteron observables at energy Elab=3MeV

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Faddeev-Noyes-Noble-Merkuriev equations in configuration space (FNNM)

L. D. Faddeev, Zh. Eksp. Teor. Fiz. 39, 1459 (1960); [Sov. Phys.-JETP 12, 1014 (1961)] *Mathematically correct manner to study three-body problem in momentum space*

Faddeev-Noyes equation in configuration space:

H.P. Noyes and H. Fiedeldey, ***in Three-Particle in Quantum Mechanics. Proceeding of the Texas A&M Conference*****, edited by J. Gillespie and J. Nuttall (Benjamin, New-York, 1968) pp.195-294.**

Three-body Problem with Charged Particles:

J.W. Noble, Phys. Rev.161 945 (1967)

Effective approach to solve three-body problem with modern NN potentials:

S.P. Merkuriev, C. Gignoux and A. Laverne, Ann. Phys. 99, 30 (1976)

FNNM equations for pnn system

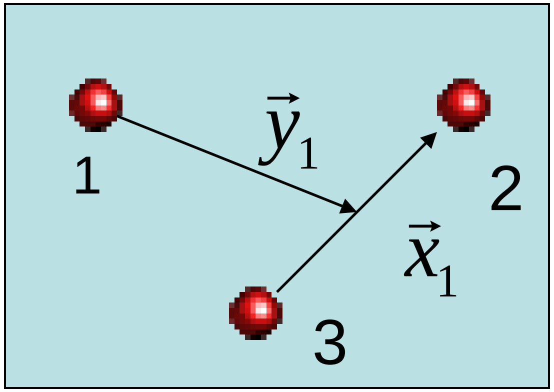
Motivation :

1. Development of an alternative method for the study of Nd-scattering using mathematically rigorous way.

2. To generalize the approach initiated by Merkuriev et al. for modern *NN* potentials (for example, the charge-dependent AV14 NN potential and others).

3. To develop a new numerical method based on the Numerov manner, to increase accuracy of calculations.

FNNM equations for ppn system



$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{3}\nabla_i^2 + V_c + \sum_{j<k} V_{jk} \ \ (+\sum_{j<k<l} V_{jkl}),$$

In this study we neglected by three-nucleon forces V_{jkl}

$$\Psi = \Phi_1 + \Phi_2 + \Phi_3 = (1 + P^+ + P^-)\Phi_1,$$

$$[-\frac{\hbar^2}{m}(\Delta_{\vec{x}_1} + \Delta_{\vec{y}_1}) + V_c + V(\vec{x}_1) - E]\Phi(\vec{x}_1, \vec{y}_1) = -V(\vec{x}_1)(P^+ + P^-)\Phi(\vec{x}_1, \vec{y}_1),$$

$$(P^+ : 123 \rightarrow 231, P^- : 123 \rightarrow 321)$$

$$V_c = \sum_{\alpha}\frac{n}{|x_{\alpha|}}\prod_{i\subset\alpha}\frac{1}{2}(1 + \tau_z^i), \qquad n = \frac{me^2}{\hbar^2},$$

FNNM equations for ppn system

In the MGL approach [S.P. Merkuriev, C. Gignoux and A. Laverne, Ann. Phys. 99,30 (1976)]

$$\left[E + \frac{\hbar^2}{m}(\partial_x^2 + \partial_y^2) - v_\alpha^{\lambda l}(x,y)\right]\Phi_\alpha^{\lambda_0,s_0,M_0}(x,y) = \sum_\beta \left[v_{1,\alpha\beta} + \sum_\tau (v_\tau^+ c_{\tau,\alpha\beta}^{M_0+} + v_\tau^- c_{\tau,\alpha\beta}^{M_0-})\right]$$

$$\times \Phi_\beta^{\lambda_0,s_0,M_0}(x,y) + \sum_\beta v_{\alpha\beta}(x)\left[\Phi_\beta^{\lambda_0,s_0,M_0}(x,y) + \int_{-1}^{1} du \sum_\gamma g_{\beta\gamma}(y/x,u)\Phi_\gamma^{\lambda_0,s_0,M_0}(x',y')\right],$$

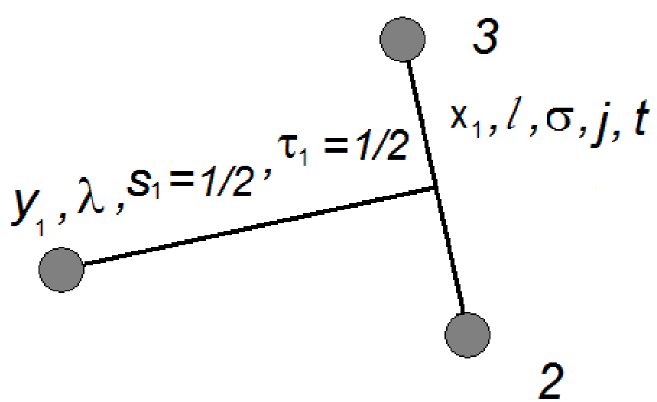
$\alpha = \{l,\sigma,j,s,\lambda,t,T\}$, s is the total "spin" $(\mathbf{s} = \mathbf{1/2} + \mathbf{J})$

M is the total three-particle momentum , $\mathbf{M} = \vec{\lambda} + \mathbf{s}$

v_1 and coefficients $c^{M_0\pm}$ depending on quantum state numbers of channel combined with $v^\pm$ are matrix elements of the Coulomb potential projected onto the MGL basis.

If $\alpha = \{l\sigma js\lambda tT\}$ and $\beta = \{l'\sigma' j' s' \lambda' t' T'\}$ then values of τ are restricted by the following inequality: $\max(|l-l'|, \lambda - \lambda'|) \le \tau \le \min(l+l', \lambda+\lambda')$

$$v_{\alpha\alpha'}(x) = \delta_{\lambda\lambda'}\delta_{ss'}\delta_{\sigma\sigma'}\delta_{JJ'}v_{ll'}^{\sigma J}$$



FNNM equations for pnn system

Regularity conditions

$$\Phi_{\alpha}^{\lambda_0 s_0 M_0}(0, \mathsf{y}) = \Phi_{\alpha}^{\lambda_0 s_0 M_0}(\mathsf{x}, 0) = \Phi_{\alpha}^{\lambda_0 s_0 M_0}(\mathsf{x}_{\mathsf{max}}, \mathsf{y}) = 0$$

Asymptotic condition for *pd* elastic scattering

incoming wave

scattered wave

$$\Phi_{1,\bar{\alpha}}^{\lambda_0 s_0 M_0}(x,y) \sim \Big\{ \underbrace{\delta_{\lambda\lambda_0}\delta_{ss_0}\delta_{\sigma 1}\delta_{j1} e^{i\Delta_\lambda^c} F_\lambda^c(qy)}_{\text{incoming wave}} + \underbrace{e^{-i\Delta_\lambda^c}\Big(G_\lambda^c(qy) + iF_\lambda^c(qy)\Big) a_{\lambda s \lambda_0 s_0}^{M_0}}_{\text{scattered wave}} \Big\} \psi_l(x),$$

$$x \text{ finite}, \quad y \to \infty,$$

where Δ_λ^c =arg$\Gamma(\lambda+1+i\nu)$ is the Coulomb phase and ν is equal $n/(\sqrt{3}q)$, ψ_l is lth component of deuteron wave function (l =0,2), and F^c and G^c are the regular and irregular Coulomb functions, respectively.

S.P. Merkuriev, Ann. Phys. (N.Y.) 130, 3975 (1980),
S.P. Merkuriev, Acta Physica (Austriaca), Suppl. XXIII, 65 (1981),
A.A. Kvitsinsky, Yu. A. Kuperin, S.P. Merkuriev, A.K. Motovilov and S.L. Yakovlev, Fiz. Elem. Chastis At. Yadra, 17, 267 (1986).

$a_{\lambda s \lambda_0 s_0}^{M_0}$

FNNM equations for pnn system

In the polar coordinates: $\rho^2 = x^2 + y^2 \qquad \tan(\theta) = y/x$

$$\left[E + \frac{\hbar^2}{m}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + \frac{1}{4\rho^2}\right) - v_\alpha^{\lambda l}(\rho,\theta)\right] U_\alpha^{\lambda_0 s_0 M_0}(\rho,\theta) = \frac{n}{\rho}\sum_\beta Q_{\alpha\beta} U_\beta^{\lambda_0 s_0 M_0}(\rho,\theta)$$

$$+\sum_\beta v_{\alpha\beta}(\rho,\theta)\left[U_\beta^{\lambda_0 s_0 M_0}(\rho,\theta) + \int_{-1}^{1} du \sum_\gamma g_{\beta\gamma}(\theta,u,\theta'(\theta,u)) U_\gamma^{\lambda_0 s_0 M_0}(\rho,\theta')\right],$$

$$\text{where} \quad \cos^2\theta'(u,\theta) = \frac{1}{4}\cos^2\theta - \frac{\sqrt{3}}{2}\cos\theta\sin\theta\cdot u + \frac{3}{4}\sin^2\theta, \quad U = \rho^{-1/2}\Phi$$

Method of solution

Discretization of the FNNM equations:
Numerov Method + spline approximation

V.M. Suslov and B. Vlahovic, Phys. Rev. C 69, 044003 (2004).

Numerov Method is used in ρ variable with accuracy $(\Delta\rho)^4$

Hermitian splines in θ variable with accuracy $(\Delta\theta)^4$

[A.A. Kvitsinsky and C.-Y. Hu, Few-Body Syst. 12, 7 (1992)]

Method of solution

Numerov Method

V.M. Suslov and B. Vlahovic, Phys. Rev. C 69, 044003 (2004).

$$-\Big[E + \frac{12}{(\Delta\rho)^2} + (1 + \frac{2\Delta\rho}{\rho_j})\frac{T_\alpha(\theta)}{\rho_j^2}\Big]U_\alpha(\rho_{j-1},\theta) + n\sum_\beta \frac{Q_{\alpha\beta}(\theta)}{\rho_j}(1 + \frac{\Delta\rho}{\rho_j})U_\beta(\rho_{j-1},\theta)$$

$$+\sum_\beta (v_{\alpha\beta}(\rho_j,\theta) - \Delta\rho v'_{\alpha\beta}(\rho_j,\theta))(U_\beta(\rho_{j-1},\theta) + \sum_\gamma \int_{\theta^-}^{\theta^+} d\theta' g_{\beta\gamma}(\theta,\theta')U\gamma(\rho_{j-1},\theta'))$$

$$-2\Big[5E - \frac{12}{(\Delta\rho)^2} + (5 + \frac{3\Delta\rho}{\rho_j})\frac{T_\alpha(\theta)}{\rho_j^2}\Big]U\alpha(\rho_j,\theta) + 2n\sum_\beta \frac{Q_{\alpha\beta}(\theta)}{\rho_j}(5 + \frac{(\Delta\rho)^2}{\rho_j^2})U_\beta(\rho_j,\theta)$$

$$+\sum_\beta (10v_{\alpha\beta}(\rho_j,\theta) + (\Delta\rho)^2 v''_{\alpha\beta}(\rho_j,\theta))(U_\beta(\rho_j,\theta) + \sum_\gamma \int_{\theta^-}^{\theta^+} d\theta' g_{\beta\gamma}(\theta,\theta')U_\gamma(\rho_j,\theta'))$$

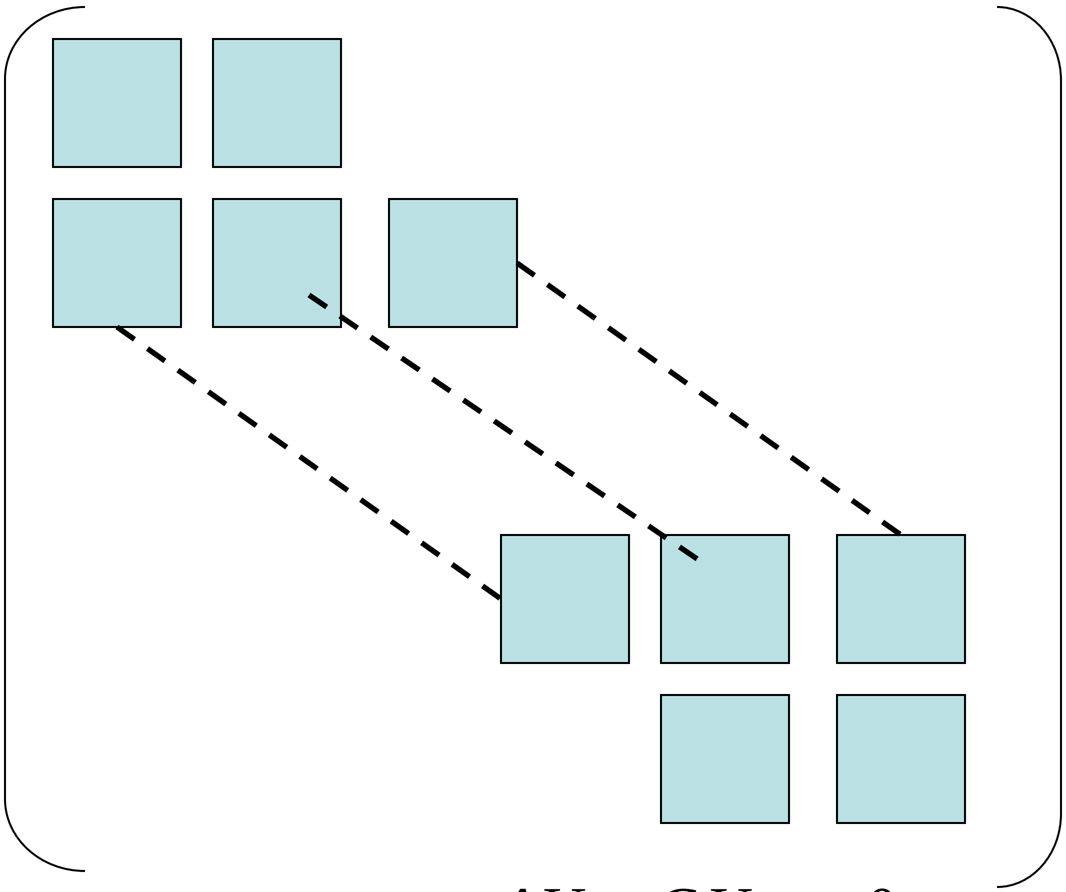
$$-\Big[E + \frac{12}{(\Delta\rho)^2} + (1 - \frac{2\Delta\rho}{\rho_j})\frac{T_\alpha(\theta)}{\rho_j^2}\Big]U_\alpha(\rho_{j+1},\theta) + n\sum_\beta \frac{Q_{\alpha\beta}(\theta)}{\rho_j}(1 - \frac{\Delta\rho}{\rho_j})U_\beta(\rho_{j+1},\theta)$$

$$+\sum_\beta (v_{\alpha\beta}(\rho_j,\theta) + \Delta\rho v'_{\alpha\beta}(\rho_j,\theta))(U_\beta(\rho_{j+1},\theta) + \sum \int_{\theta^-}^{\theta^+} d\theta' g_{\beta\gamma}(\theta,\theta')U_\gamma(\rho_{j+1},\theta')),$$

$$\text{where} \quad T_\alpha(\theta) = \frac{\partial^2}{\partial\theta^2} - \frac{l(l+1)}{\cos^2\theta} - \frac{\lambda(\lambda+1)}{\sin^2\theta} + \frac{1}{4}.$$

Method of solution

Matrix structure:



$$\begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_{N-1} \\ U_N \end{pmatrix} = -\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ G_N U^{as} \end{pmatrix}$$

$$\begin{aligned} A_1 U_1 + G_1 U_2 &= 0 \\ B_j U_{j-1} + A_j U_j + G_j U_{j+1} &= 0,\ j = 2,\ldots,N\text{-}1 \\ B_N U_{N-1} + A_N U_N &= -G_N U_{N+1} = -G_N U^{as} \end{aligned}$$

Method of solution

$$(D * U)_i = -\delta_{iN_\rho} G_{N_\rho} U_{N_\rho+1},$$

matrix D is of dimension $N_\rho N_{in} \times N_\rho N_{in}$, $N_{in} = N_\alpha N_c$

$$U_j = -D^{-1}_{jN_\rho} G_{N_\rho} U_{N_\rho+1}, \quad j = 1, 2....N_\rho.$$

Consider the last component of vector U:

$$U_{N_\rho} \quad = -D^{-1}_{N_\rho N_\rho} G_{N_\rho} U_{N_\rho+1}.$$

Provided R_{max} is large enough, we obtain set of linear equations for the unknown amplitudes $a^{M_0}_{\lambda s \lambda_0 s_0}$:

$$\sum_{i=1}^{3} a^{M_0}_{ij} \cdot \mathbf{v}^i = \mathbf{F}^j, \quad j = 1, 2, 3.$$

Method of solution

Least Squares Method:

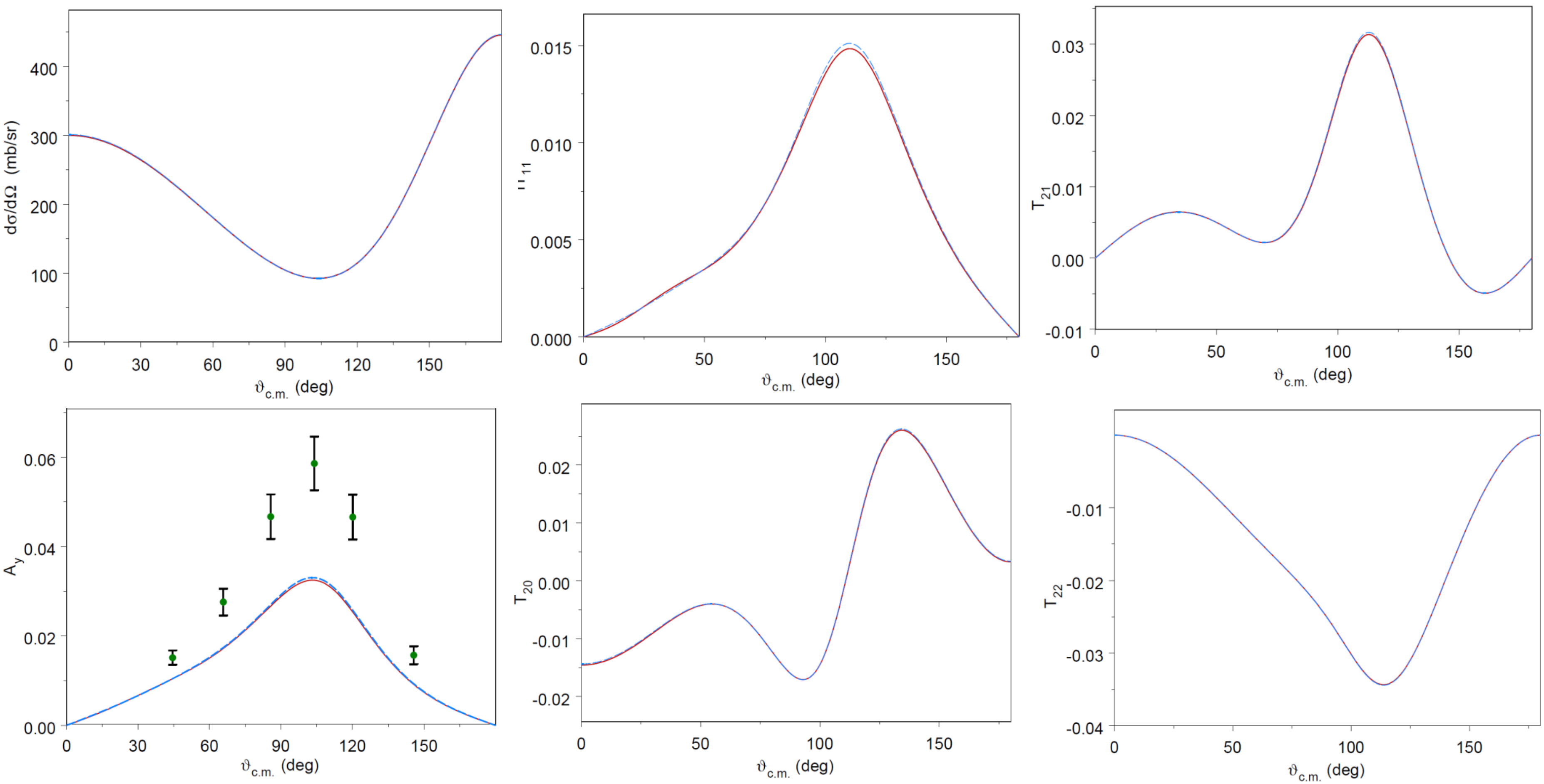
$$a_{1j}\begin{pmatrix} V_1^1 \\ \vdots \\ V_N^1 \end{pmatrix} + a_{2j}\begin{pmatrix} V_1^2 \\ \vdots \\ V_N^2 \end{pmatrix} + a_{3j}\begin{pmatrix} V_1^3 \\ \vdots \\ V_N^3 \end{pmatrix} = \begin{pmatrix} F_1^j \\ \vdots \\ F_N^j \end{pmatrix}, j = 1,2,3$$

$$\| \sum_{i=1}^{3} a_{ij}^{M_0} \cdot \mathbf{v}^i - \mathbf{F}^j \|^2 = \min .$$

$$\sum_{i=1}^{3} a_{ij}^{M_0} \cdot (\mathbf{v}_k^*, \mathbf{v}^i) = (\mathbf{v}_k^*, \mathbf{F}^j), \quad j = 1, 2, 3, \quad k = 1, 2, 3,$$

Elastic Scattering Observables

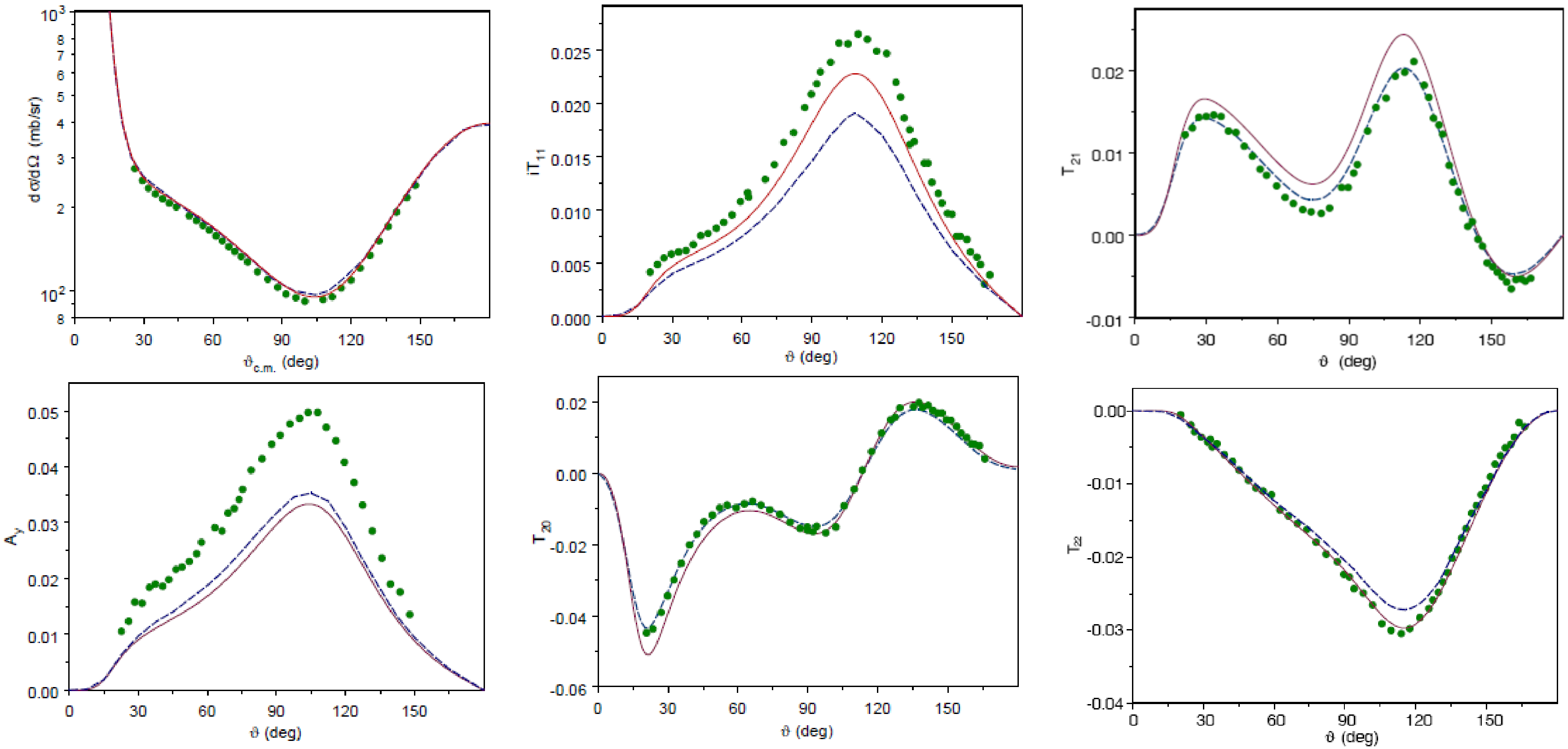
neutron-deuteron scattering at E_{lab} = 3 MeV



The solid lines correspond to our results obtained with AV14 NN potential. The dashed lines correspond to A. Kievsky et al. results obtained with AV14 NN potential (Phys. Rev. C 58, 3085 (1998)). The experimental data are from J.E. McAninch et al. (Phys. Lett. B 307, 13 (1993)).

Elastic Scattering Observables

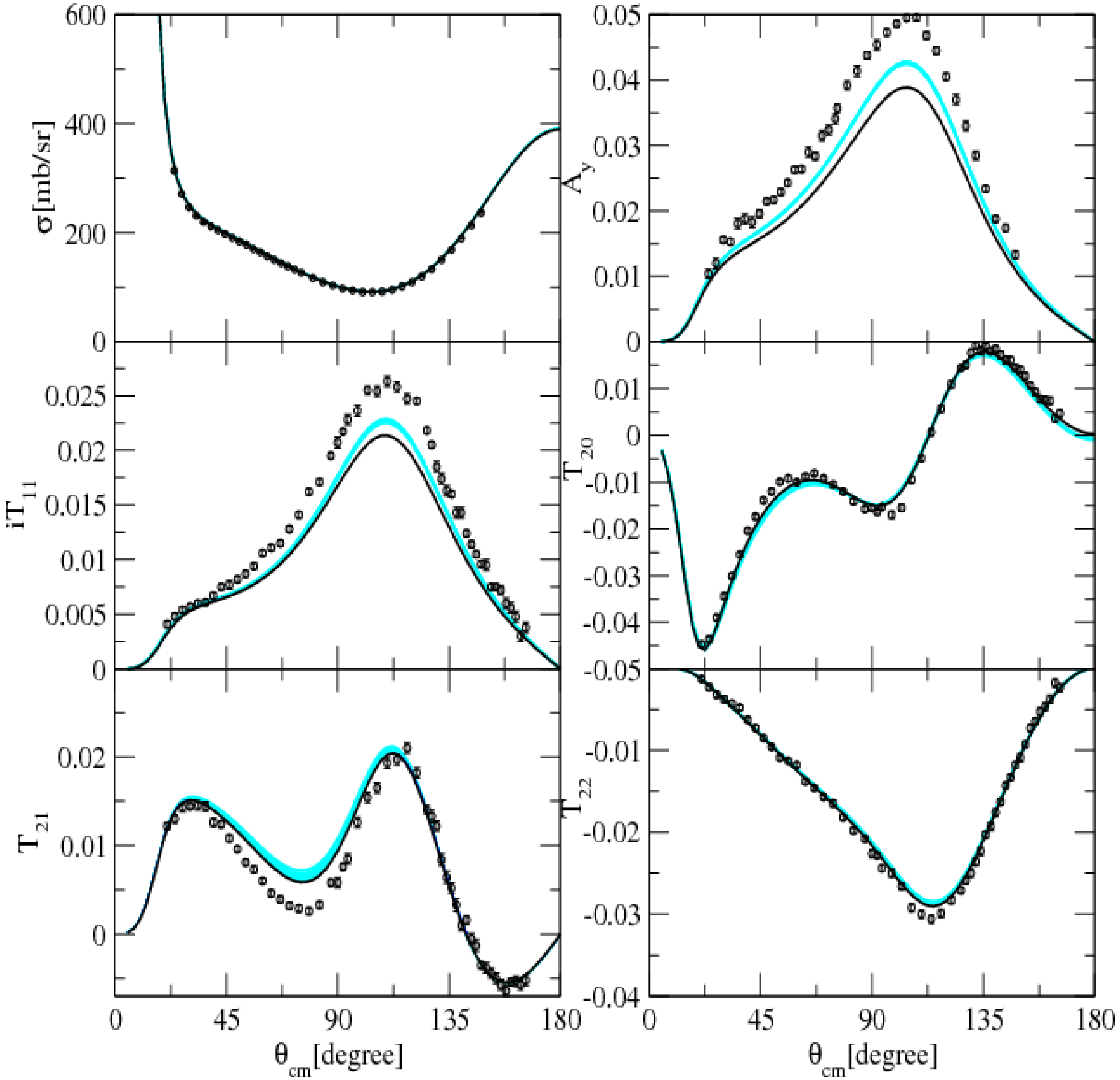
proton-deuteron scattering at E_{lab} = 3 MeV



The solid lines correspond to our results obtained with AV14 NN potential. The dashed lines correspond to A. Deltuva et al. (Phys. Rev. C 71, 064003-1 (2005)) results obtained with AV18 NN potential.
The experimental data are from S. Shimizu et al. (Phys. Rev. C 52, 1193 (1995)).

Comparative study: A.Kievsky at al.

arXiv:1002.1601v1 [nucl-th] 2010



Differential cross section and vector and tensor polarization observables at $E_{lab} = 3$ MeV

using the AV18+N2LOL model with the parameters given in Table IV (cyan band).

The predictions of the AV18+URIX model (solid line) and the experimental points are also shown.

Conclusions

- *Very good agreement between predictions of our calculations and those of benchmark calculations demonstrates the soundness of our novel method providing thereby a new approach for calculating three-nucleon scattering including strong and electromagnetic interactions.*

- *Our approach can and will be used to include three nucleon forces .*

- *Our preliminary goal is to extend our approach using AV14 NN potential and including the Coulomb potential to energies above the two-body threshold and to focus on breakup data and on established discrepancies.*

- *Next step is to use the AV18 NN potential. As discussed in this report, we have already established interesting differences in iT_{11}, T_{20} and T_{22} most likely due to difference between AV14 and AV18 NN potentials.*

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